

DIGITAL SIGNAL PROCESSING

[16EE513]

FIFTH SEMESTER

2016 REGULATION

PREPARED BY

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

K.S.R. COLLEGE OF ENGINEERING

(AN AUTONOMOUS INSTITUTION, AFFILIATED TO ANNA UNIVERSITY CHENNAI)

K.S.R. KALVI NAGAR, TIRUCHENGODE – 637 215

NAMAKKAL (DT), TAMILNADU, INDIA.

| K.S.R. COLLEGE OF ENGINEERING (Autonomous) | | | | R 2016 | | | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------|--|--|--------|---|---|---------------|
| <u>SEMESTER - V</u> | | | | | | | |
| 16EE513 | DIGITAL SIGNAL PROCESSING | | | L | T | P | C |
| | | | | 3 | 1 | 0 | 4 |
| Prerequisite: Knowledge on Engineering Mathematics and Signals and Systems. | | | | | | | |
| Objective: To acquaint the students with Discrete Time System Analysis, Filter Designs and features of TMS processors. | | | | | | | |
| UNIT - I | DISCRETE TIME SYSTEM ANALYSIS | | | | | | [12] |
| Need and advantages of Digital Signal Processing – Typical DSP System: Sampling, Quantization, Quantization Error, Nyquist rate, Aliasing effect – Z-Transform and ROC – properties of Z-Transform – Inverse Z-Transform – Solution of Difference Equation using Z-Transform – Stability Analysis – Convolution using Z-Transform. | | | | | | | |
| UNIT - II | DISCRETE FOURIER TRANSFORM | | | | | | [12] |
| DFT: Definition and its properties – Computation of DFT and IDFT – Computation of DFT using DIT and DIF-FFT Radix 2 algorithms – Computation of IDFT using DIT and DIFFFT algorithms. | | | | | | | |
| UNIT - III | DESIGN OF IIR FILTERS | | | | | | [12] |
| Realization of IIR filter: Direct form I and II, cascade and parallel forms – Analog low pass filter design: Butterworth and Chebyshev – Digital filter design: Impulse invariant method and Bilinear transformation – Warping, prewarping. | | | | | | | |
| UNIT – IV | DESIGN OF FIR FILTERS | | | | | | [12] |
| Amplitude and Phase response of FIR filters – Linear phase characteristics – Design of FIR filters using windows: Rectangular, Triangular, Hamming and Hanning. | | | | | | | |
| UNIT - V | DSP HARDWARE | | | | | | [12] |
| Introduction – Selection of DSP processor – Application of DSP processor – Van Neumann architecture – Harvard architecture – TMS320C50 digital signal processor: Architecture, Addressing modes and Instruction set. | | | | | | | |
| Total = 60 Periods | | | | | | | |
| Course Outcomes: | | | | | | | |
| CO1. Analyze the discrete time system and the signal processing through DSP systems. | | | | | | | |
| CO2. Solve the problems on LTI using Discrete Fourier Transform. | | | | | | | |
| CO3. Formulate and design FIR filters using various windowing functions. | | | | | | | |
| CO4. Formulate and design IIR filters using analog and digital filter designs. | | | | | | | |
| CO5. Explain the architecture, addressing modes and instruction sets of digital signal processors and its application. | | | | | | | |
| Text Books : | | | | | | | |
| 1 | Anand Kumar.A, Digital Signal Processing, PHI, Second Edition, 2015. | | | | | | |
| 2 | John G.Prokis, Dimtris G. Manolakis, Digital Signal Processing Principles, Algorithms and Application, PHI, Fourth Edition, 2011. | | | | | | |
| Reference Books : | | | | | | | |
| 1 | Alan V. Oppenheim, Ronald W. Schafer, John R.Back, Discrete Time Signal Processing, PHI, Second Edition, 2008. | | | | | | |
| 2 | Johnny R.Johnson, Introduction To Digital Signal Processing, Prentice Hall, 2009. | | | | | | |
| 3 | Mitra.S.K, Digital Signal Processing – A computer based approach, Tata McGraw-Hill, Fourth Edition, 2013. | | | | | | |
| 4 | Salivanan.S, Vallavaraj.A, Gnanapriya.C, Digital Signal Processing, Tata McGraw Hill, Second Edition, 2011. | | | | | | |

K.S.R COLLEGE OF ENGINEERING, TIRUCHENGODE - 637 215
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

LESSON PLAN

III YEAR - EEE (2018 -19)

Subject : 16EE513 - Digital Signal Processing

Year / Sem: III / V

Regulation : 2016

Staff Name: Dr.C.Gowri Shankar

Mr.J.Thiyagarajan

| LECTURE | TOPIC | HOURS | TEACHING AID | SOURCE | PAGE. NO |
|--------------------------------------|----------------------------------------------------------------|-----------|--------------|--------|-------------|
| UNIT - I | | | | | |
| DISCRETE TIME SYSTEM ANALYSIS | | | | | |
| 1 | Introduction to Signals and Systems | 1 | BB | R - 1 | 1.22 |
| 2 | Need & Advantages of DSP | 1 | „ | R - 1 | 1.20 |
| 3 | Typical DSP System: Sampling, Quantization, Quantization Error | 1 | „ | R - 1 | 1.168 |
| 4 | Nyquist rate, Aliasing effect | 1 | „ | R - 1 | 1.70 |
| 5 | Z-Transform and ROC | 1 | „ | R - 1 | 2.1 |
| 6 | Properties of Z-Transform | 1 | „ | R - 1 | 2.8 |
| 7 | Inverse Z-Transform | 1 | „ | R - 1 | 2.30 |
| 8 | Solution of Difference Equation using Z-Transform | 1 | „ | R - 1 | 2.52 |
| 9 | Stability Analysis – Convolution using Z-Transform. | 1 | „ | R - 1 | 2.26 & 2.58 |
| 10 | Tutorial | 3 | „ | | |
| Total | | 12 | | | |
| UNIT - II | | | | | |
| DISCRETE FOURIER TRANSFORM | | | | | |
| 11 | DFT: Definition | 1 | BB | R - 1 | 3.5 |
| 12 | DFT: Properties | 1 | „ | R - 1 | 3.25 |
| 13 | Computation of DFT and IDFT | 1 | „ | R - 1 | 4.1 |
| 14 | Computation of DFT using DIT FFT Radix 2 algorithms | 2 | „ | R - 1 | 4.3 |
| 15 | Computation of DFT using DIF FFT Radix 2 algorithms | 2 | „ | R - 1 | 4.21 |
| 16 | Computation of IDFT using DIT FFT algorithms | 1 | „ | R - 1 | 4.33 |
| 17 | Computation of IDFT using DIF FFT algorithms | 1 | „ | R - 1 | 4.33 |
| 18 | Tutorial | 3 | „ | | |
| Total | | 12 | | | |

| LECTURE | TOPIC | HOURS | TEACHING AID | SOURCE | PAGE. NO |
|---------------------------------------------------|---------------------------------------------------------|-----------|--------------|--------|----------|
| UNIT - III DESIGN OF IIR FILTERS | | | | | |
| 19 | Realization of IIR filter: Direct form I and II | 1 | BB | R - 1 | 5.54 |
| 20 | Cascade and Parallel forms | 1 | „ | R - 1 | 5.65 |
| 21 | Analog low pass filter design: Butterworth Filter | 2 | „ | R - 1 | 5.6 |
| 22 | Chebyshev filter design | 2 | „ | R - 1 | 5.17 |
| 23 | Digital filter design using Impulse invariant method | 1 | „ | R - 1 | 5.35 |
| 24 | Digital filter design using Bilinear transformation | 1 | „ | R - 1 | 5.44 |
| 25 | Warping & Prewarping | 1 | „ | R - 1 | 5.6 |
| 26 | Tutorial | 3 | | | |
| Total | | 12 | | | |
| UNIT - IV DESIGN OF FIR FILTERS | | | | | |
| 27 | FIR filters: Amplitude and Phase response | 1 | BB | R - 1 | 6.1 |
| 28 | Linear phase characteristics | 1 | „ | R - 1 | 6.5 |
| 29 | Design of FIR filters using windows: Rectangular Window | 1 | „ | R - 1 | 6.32 |
| 30 | Triangular Window | 2 | „ | R - 1 | 6.35 |
| 31 | Hamming Window | 2 | „ | R - 1 | 6.40 |
| 32 | Hanning Window | 2 | „ | R - 1 | 6.39 |
| 33 | Tutorial | 3 | | | |
| Total | | 12 | | | |
| UNIT - V DSP HARDWARE | | | | | |
| 34 | Selection of DSP processor | 1 | PPT | R - 1 | 11.5 |
| 35 | Application of DSP processor | 2 | „ | R - 1 | 11.6 |
| 36 | Van Neumann architecture | 2 | „ | R - 1 | 11.8 |
| 37 | Harvard architecture | 2 | „ | R - 1 | 11.9 |
| 38 | Architecture of TMS320C50 processor | 1 | „ | R - 1 | 11.15 |
| 39 | Addressing modes of TMS320C50 processor | 2 | „ | R - 1 | 11.25 |
| 40 | Instruction set of TMS320C50 processor | 2 | „ | R - 1 | 11.28 |
| Total | | 12 | | | |

Teaching Aid:

1. BB - Black Board.
2. LCD - LCD Projector.
3. OHP - Over Head Projector.
4. Media - Multimedia.
5. Model - Physical Mode.

Reference Books:

- R1) Ramesh Babu P, **Digital Signal Processing**, Scitech Publications (India) Pvt Ltd, Sixth Edition, 2016.
- R2) Anand Kumar A, **Digital Signal Processing**, Prentice Hall India Pvt Ltd, First Edition, 2013.
- R3) John G Proakis, Dimtris G Manolakis, **Digital Signal Processing Principles, Algorithms and Application**, PHI, 3rd Edition, 2000,
- R4) B.Venkataramani & M. Bhaskar, **Digital Signal Processor Architecture, Programming and Application**, TMH 2002.
- R5) Alan V Oppenheim, Ronald W Schafer, John R Back, **Discrete Time Signal Processing**, PHI, 2nd Edition, 2000.
- R6) S.K.Mitra, '**Digital Signal Processing- A Computer based approach**', Tata McGraw-Hill, New Delhi, 1998.
- R7) S.Salivahanan, A.Vallavaraj, C.Gnanapriya, '**Digital Signal Processing**', Tata McGraw Hill, New Delhi, 2003.

Faculty Signature**HOD/EEE**

UNIT - I

DISCRETE TIME SYSTEM ANALYSIS

1. Define signal. (Remembering)

A signal is defined as any physical quantity that varies with time, space or any other independent variable or variables.

2. What is multiple dimensional signals? Give Examples. (Remembering)

A signal which is a function of two or more independent variables is called multi dimensional signal.

3. What is analog signal? (Remembering)

The analog signal is a continuous function of an independent variable such as time, space, etc. the analog signal is defined for every instant of the independent variable and so the magnitude of analog signal is continuous in the specified range.

4. Define system? (Remembering)

A system is defined as a physical device that generates a response or an output signal, for a given input signal.

5. Define continuous time signals. (Remembering)

The signals that are defined for every instant of time are known as continuous time signals. They are denoted by $x(t)$.

6. Define discrete time signals. (Remembering)

The signals that are defined at discrete instants of time are known as discrete time signals. The discrete time signals are continuous in amplitude and discrete in time. They are denoted by $x(n)$.

7. What is Digital signal processing (DSP)? (Remembering)

The DSP refers to processing of signals by digital systems.

8. What are the advantages of DSP? (Remembering)

The advantages of DSP are

- i) The programs can be modified easily for better performance.
- ii) Better accuracy can be achieved by using adaptive algorithm.
- iii) The digital signals are easily stored and transported.
- iv) The digital systems are cheaper than analog equivalent.

9. Give some application of DSP? (Remembering)

1. Speech – Speech Compression & decompression for voice storage system and for transmission and reception of voice signals.
2. Communication – Elimination of noise by filtering and echo cancellation by adaptive Filtering in transmission channels.
3. Biomedical – Spectrum analysis of ECG, EE, etc., signals to identify various disorders in heart, brain, etc.

10. What are the various methods of representing discrete time signal? (Remembering)

The various method of representing discrete time signals are

1. Functional Representation.
2. Graphical Representation.
3. Tabular Representation.
4. Sequence Representation.

11. Give the classification of signals? (Remembering)

- Continuous-time and discrete time signals
- Even and odd signals
- Periodic signals and non-periodic signals
- Deterministic signal and Random signal
- Energy and Power signal

12. What are the types of systems? (Remembering)

- Linear and Non-linear systems
- Causal and Non-causal systems
- Static and Dynamic systems
- Time varying and time in-varying systems
- Distributive parameters and Lumped parameters systems
- Stable and Un-stable system
- FIR and IIR systems.

13. What are even and odd signals? (Remembering)

Even signal: continuous time signal $x(n)$ is said to be even if it satisfies the condition

$$x(n) = x(-n) \text{ for all values of } n.$$

Odd signal: The signal $x(n)$ is said to be odd if it satisfies the condition $x(-n) = -x(n)$ for all n .

In other words even signal is symmetric about the time origin or the vertical axis, but odd signals are anti-symmetric about the vertical axis.

14. What are deterministic and random signals? (Remembering)

Deterministic Signal:

A deterministic signal is a signal about exhibiting no uncertainty of value at any given instant time. Its instantaneous value can be accurately predicted by specifying a formula, algorithm or simply by its describing statement in words.

Example: $v(t) = A_0 \sin \omega(t)$

Random signal:

Random signal is a signal characterized by uncertainty before its actual occurrence. Such signal may be viewed as group of signals with each signal in the ensemble having different wave forms.(e.g.) The noise developed in a television or radio amplifier is an example for random signal.

15. What are energy and power signal? (Remembering)

Energy signal:

The Energy of a discrete time signal $x(n)$ is defined as

$$\text{Energy (E)} = \sum_{n=-\infty}^{\infty} [x(n)]^2$$

A signal $x(n)$ is called an Energy signal if and only if the energy obeys the relation $0 < E < \infty$. For an Energy Signal $P = 0$.

Power Signal:

The Average power of discrete time signal $x(n)$ is defined as

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N [x(n)]^2$$

A signal is power signal, if and only if the total energy of the signal is finite. For an energy signal $P = 0$. Similarly the signal is said to be power if the average power of the signal is finite. For a power signal $E = \infty$. The signals that not satisfy above properties are neither energy nor power signals.

16. What are the operations performed on a signal? (Remembering)**Operations performed on dependent variables:**

Amplitude scaling: $y(t) = c x(t)$, where c is the scaling factor, $x(t)$ is the continuous time signal.

Addition: $y(t) = x_1(t) + x_2(t)$

Multiplication $y(t) = x_1(t) x_2(t)$

Differentiation: $y(t) = d/dt x(t)$

Integration $y(t) = \int x(t) dt$

Operations performed on independent variables

Time shifting,

Time scaling

Time reversal,

Scalar Multiplication

Signal Multiplier,

Addition Operation

Sampling and aliasing

17. What are elementary signals and name them? (Remembering)

The elementary signals serve as a building block for the construction of more complex signals. They are also important in their own right, in that they may be used to model many physical signals that occur in nature.

There are five elementary signals. They are as follows

Unit step function

Unit impulse function

Ramp function

Exponential function

Sinusoidal function

18. What is memory system and memory less system? (Remembering)

A system is said to be memory system if its output signal at any time depends on the past values of the input signal. Circuit with inductors capacitors are examples of memory system.

A system is said to be memory less system if the output at any time depends on the present values of the input signal. An electronic circuit with resistors is an example for memory less system.

19. What is an invertible system? (Remembering)

A system is said to be invertible system if the input of the system can be recovered from the system output. The set of operations needed to recover the input as the second system connected in cascade with the given system such that the output signal of the second system is equal to the input signal applied to the system.

$$H^{-1}\{y(t)\} = H^{-1}\{H\{x(t)\}\}.$$

20. What are time invariant systems? (Remembering)

A system is said to be time invariant system if a time delay or advance of the input signal leads to an identical signal. This implies that a time invariant system responds identically no matter when the input signal is applied. It also satisfies the Condition

$$R\{x(n-k)\} = y(n-k).$$

21. What is continuous time system? (Remembering)

A continuous time system is one which operates on a continuous time signal and produces Continuous time output signal. If the input and output of continuous time systems are $x(t)$ and $y(t)$. Then we say that $x(t)$ is transformed to $y(t)$.

$$y(t) = T[x(t)]$$

22. What is discrete time system? (Remembering)

A discrete time system is one which operates on a discrete – time signal and produces a discrete - time output signal. If the input and output of discrete time system are $x(n)$ and $y(n)$, then we can write

$$y(n) = T[x(n)]$$

23. Define signal processing. (Remembering)

Signal processing is any operation that changes the characteristics of a signal. These characteristics include the amplitude, shape, phase and frequency content of the signal.

24. What is causal and non causal signals. (Remembering)

A signal $x(n)$ is said to be causal if its value is zero for $n < 0$. Otherwise the signal is non causal.

Example for an causal signals.

$$x_1(n) = a^n u(n)$$

$$x_2(n) = \{1,2,3,4\}$$

Example for non causal Signals

$$X_1(n) = a^n u(-n+1)$$

25. What Linear and Non linear systems? (Remembering)

A system that satisfies the superposition principle is said to be a linear system. Super position principle state that the response of the system to a weighted sum of signals should be equal to the corresponding weighted sum of the outputs of the system to each on the individual input signals.

$$T [a_1x_1 (n) + a_2 x_2 (n)] = a_1 T[x_1 (n)] + a_2 T[x_2 (n)]$$

For any arbitrary constants a_1 and a_2 .

A relaxed system that does not satisfy the superposition principle is called non linear.

26. Write the properties of convolution. (Remembering)

i) Commutative law: $x (n) * h (n) = h (n) * x (n)$

ii) Associative law: $[x (n)*h_1 (n) * h_2 (n) = x (n) * [h_1 (n) * h_2 (n)]$

iii) Distributive law: $x (n) * [h_1 (n) + h_2 (n)] = x (n) * h_1 (n) + x (n) * h_2 (n)$

27. Define FIR System. (Remembering)

If the impulse response of the system is of finite duration, then the system is called a finite impulse response (FIR system).

Example

$$h (n) = \begin{cases} 1 & \text{for } n = -1, 2 \\ 2 & \text{for } n = 1 \\ 3 & \text{for } n = 0, 3 \\ 0 & \text{otherwise.} \end{cases}$$

28. Define IIR System. (Remembering)

An Infinite Impulse response (IIR) system has an impulse response for infinite duration An example of an IIR system is $h (n) = a^n u (n)$

29. What is the condition for system stability? (Remembering)

The necessary and sufficient condition guaranteeing the stability of a linear time – invariant system is that its impulse response is absolutely sumable.

$$\sum_{k=-\infty}^{\infty} [h(K)] < \infty$$

30. State sampling theorem. (Remembering)

A band limited continuous time signal, with higher frequency f_m hertz, can be uniquely recovered from its samples provided the sampling rate $F \geq 2f_m$ samples per second.

31. How will you generate a ramp signal? (Remembering)

The ramp signal can be generated from the mathematical expression $x(n) = n$ for $n \geq 0$.

32. What is recursive and non recursive system? Give examples. (Remembering)

If the output $y(n)$ at a time n of a system depends on past output values then the system is called recursive system. The recursive systems are function of past outputs, present and past inputs.

If the output $y(n)$ at time n of a system does not depend on past output values then the system is called non recursive systems are function of present and past inputs.

Example of Recursive system: $y(n) = 0.5y(n-1) + 0.75x(n) + a x(n-1)$

Example of nonrecursive system: $y(n) = 2x(n) + 1.5x(n-1) + 0.5x(n-2)$

33. What is the function of analog to digital conversion? (Remembering)

Most of the signals encountered in nature are continuous in time. But in digital signal processing the signals are sampled and quantized at discrete – time instants and are represented as a sequence of 1 s and 0 s. This can be done by an analog to digital converter.

34. Define sampling. (Remembering)

Sampling is a process of converting a continuous – time signal into a discrete time signal.

35. Define Quantization. (Remembering)

The process of converting discrete time continuous amplitude signals $x(n)$ into a discrete time discrete amplitude signal $x_q(n)$ is known as quantization. This is done by rounding off each sample in $x(n)$ to the nearest quantization level. The difference between adjacent levels or the quantization step in terms of range of the signal is

$$Q = \text{Range of signal} / \text{Number of quantization}$$

36. Write the different types of Analog to digital converters. (Remembering)

The different types of A/D Converters are

1. Counting A/D converter
2. Successive A/D converter
3. Flash A/D converter
4. over sampling sigma – delta A/D converter

37. What is aliasing effect? (Remembering)

Let us consider a band limited signal $x(t)$ having no frequency component for $|\Omega| > \Omega_m$. If we sample the signal $x(t)$ with sampling frequency $F < 2f_m$, the periodic continuation of $X(j\Omega)$ results in spectral overlap. In this case, the spectrum $X(j\Omega)$ cannot be recovered using a low pass filter. This effect is known as aliasing effect.

38. What are the steps involved in digital signal processing (or) draw the general block diagram to show the schematic representation of DSP system. (Remembering)

The steps involved in DSP are,

- (i) Converting Analog signal to Digital signal, which is performed by A/D converter
- (ii) Processing the Digital signal by Digital system
- (iii) Converting the Digital output signal from the Digital system to analog signal using D/A converter.

39. Define a digital signal. (Remembering)

A digital signal is a special form of discrete time signal, which is discrete in both time and amplitude obtained by quantizing each value of discrete time signals.

40. Define any two basic discrete-time signals. (Remembering)

- (i) Unit impulse sequence

The unit impulse or unit sample sequence is defined as

$$\delta(n) = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{for } n \neq 0 \end{cases}$$

- (ii) Unit step sequence

The unit step sequence $u[n]$ is defined as

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

- (iii) Unit ramp sequence:

The unit ramp sequence $r[n]$ is defined as

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or equivalently $r[n] = n u[n]$

41. Define the Z Transform. (Remembering)

The Z Transform of a discrete time signal $x(n)$ is defined as

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

Where Z is a complex variable. In polar form Z can be expressed as $Z = r e^{j\omega}$,
 r is the radius of the circle.

42. What are the properties of the Z transform. (Remembering)

- | | |
|----------------------------------------------|--------------------------------|
| 1. Linearity | 2. Complex convolution theorem |
| 3. Time shift or translation | 4. Parseval's theorem |
| 5. Multiplication by an exponential sequence | 6. Initial value theorem |
| 7. Time reversal | 8. Final value theorem |
| 9. Differentiation of $X(Z)$ | 10. Correlation |
| 11. Convolution theorem | |

43. What is meant by region of convergence (ROC)? (Remembering)

The region of convergence (ROC) of $X(Z)$ is the set of all values of Z for which $X(z)$ attains a finite value.

44. Explain the linearity property of the Z transform. (Understanding)

$$\text{If } Z\{x_1(n)\} = X_1(Z) \text{ and } Z\{x_2(n)\} = X_2(Z) \quad \text{then}$$

$$Z\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 X_1(Z) + a_2 X_2(Z)$$

45. What are the properties of region of convergence? (Remembering)

1. The Region of convergence is a ring or disk in the Z plane centered at the origin.
2. The Region of convergence cannot contain any poles.
3. The Region of convergence of an LTI stable system contains the unit circle.
4. The Region of convergence must be a connected region.

46. What are the different methods of evaluating inverse Z transform? (Remembering)

The inverse Z transform can be evaluated using several methods.

1. Long Division Method
2. Partial fraction expansion method.
3. Residue method.
4. Convolution Method

47. What is SISO system and MIMO system? (Remembering)

A control system with single input and single output is referred to as single input single output system. When the number of plant inputs or the number of plant outputs is more than one the system is referred to as multiple input output system. In both the case, the controller may be in the form of a digital computer or microprocessor in which we can speak of the digital control systems.

48. Define Right hand sequence. (Remembering)

A right hand sequence is one for which $x(n) = 0$ for all $n < n_0$ Where n_0 is positive or negative but finite. If n_0 is greater than or equal to zero, the resulting sequence is causal or a positive time sequence. For such type of sequence the ROC is entire Z plane except $Z = 0$.

49. Define Left hand sequence. (Remembering)

A left hand sequence $x(n)$ is one for which $x(n) = 0$ for all $n \geq n_0$, Where n_0 is positive or negative but finite. If $n_0 \leq 0$ the resulting sequence is anticausal sequence. For such type of sequence is anticausal sequence the ROC is entire Z plane except at $Z = \infty$.

50. Define two - sided Sequence. (Remembering)

A signal that has finite duration on both the left and right hand sides is known as two sided sequence. For such type of sequence the ROC is entire Z plane Except at $Z = 0$ and $Z = \infty$.

51. State Convolution Theorem. (Remembering)

If $X(z) = Z\{x(n)\}$, and $H(z) = Z\{h(n)\}$, Then

$$Z\{x(n) * h(n)\} = X(z) H(z)$$

When $x(n) * h(n)$ Denotes the linear convolution of sequences.

52. State Parseval's relation. (Remembering)

Consider two complex sequences $x_1(n)$ and $x_2(n)$. Parseval's Relation states that

$$\sum_{n=-\infty}^{\infty} x_1(n)x_2^*(n) = \frac{1}{2\pi j} \oint_c x_1(\nu)x_2^*\left(\frac{1}{\nu^*}\right)\nu^{-1}d\nu$$

Where the counter of integration must be in the overlap of the regions of convergence of $X_1(\nu)$ and $x_2^*\left(\frac{1}{\nu^*}\right)$.

53. State Complex Convolution Theorem. (Remembering)

The Z transform of the product of two sequences is related to the Z transforms of the individual sequences through the complex convolution theorem. This theorem states that if

$$x_3(n) = x_1(n)x_2(n)$$

$$X_3(Z) = \frac{1}{2\pi j} \oint_c x_1(\nu)x_2^*\left(\frac{Z}{\nu}\right)\nu^{-1}d\nu$$

The convergence region for $X_3(Z)$ consists of all Z such that if ν is in the region of convergence for $X_1(Z)$, Then $\frac{Z}{\nu}$ is in the region of convergence for $X_2(Z)$. The contour of integration C is a closed contour inside the intersection of the convergence regions for $X_1(\nu)$ and $X_2\left(\frac{Z}{\nu}\right)$.

54. Explain the time shifting property of the z - transform. (Remembering)

$$\text{If } Z\{x(n)\} = X(Z), \text{ Then } Z\{x(n-k)\} = Z^{-k}X(Z)$$

55. Explain the scaling property of the Z - transform. (Remembering)

$$\text{If } Z\{x(n)\} = X(Z) \text{ ROC: } r_1 < |z| < r_2, \text{ then } Z\{a^n x(n)\} = X(a^{-1}Z)$$

$$\text{ROC: } |a|r_1 < Z < |a|r_2$$

56. State the initial value theorem and final value theorem. (Remembering)

Initial value theorem: If $x(n)$ is causal, then

$$x(0) = \lim_{z \rightarrow \infty} X(z)$$

Final Value Theorem: If $x(n)$ is causal, $Z[x(n)] = X(z)$, Where the ROC for $X(z)$ includes, but is not necessarily confirmed to $|z| > 1$ and $(z-1)X(z)$ has no poles on or outside the unit circle, then

$$x(\infty) = \lim_{z \rightarrow \infty} (z-1)X(z)$$

57. Explain the time reversal property of the Z transform (Remembering)

If $Z\{x(n)\} = X(z)$ roc: $r_1 < |z| < r_2$ then

$$Z\{x(-n)\} = X(Z^{-1}) \text{ ROC: } \frac{1}{r_2} < |z| < \frac{1}{r_1}$$

58. An LTI system with the system function $H(z)$ is BIBO stable if and only if the ROC for $H(z)$ contains the unit circle. True / False. (Remembering)

True

59. Explain the convolution Property of the Z - Transform. (Understanding)

If $Z\{x_1(n)\} = X_1(z)$ and $Z\{x_2(n)\} = X_2(z)$, then

$$Z\{x_1(n) * x_2(n)\} = X_1(z)X_2(z)$$

60. Define system function. (Remembering)

Let $x(n)$ and $y(n)$ is the input and output sequence of an LTI system with impulse response $h(n)$. Then the system function of the LTI system is defined as the ratio of $Y(z)$ and $X(z)$, i.e

$$H(z) = \frac{Y(z)}{X(z)}$$

Where $Y(z)$ is the Z - Transform of the output signal $y(n)$ and $X(z)$ is the z - transform of the input signal $x(n)$.

61. An LTI system having function $H(z)$ is stable if and only if all the poles of $H(z)$ are inside the unit circle. True / false. (Remembering)

True.

62. Explain about the roc of causal and anti-causal infinite sequences? (Understanding)

For causal system the roc is exterior to the circle of radius r.

For anti causal system it is interior to the circle of radius r.

63. Explain about the roc of causal and anti causal finite sequences(Understanding)

For causal system the roc is entire z plane except z=0.

For anti causal system it is entire z plane except z=∞.

64. State and prove initial value theorem of z transform. (Remembering)

If x (n) is causal then $x(0) = \lim_{z \rightarrow \infty} x(z)$

Proof:
$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \text{ ----- (1)}$$

In (1) put $n = 0 \rightarrow x(n) \rightarrow x(z) = \infty$. Hence proved

65. Determine z transform and roc of the signal {1, 2, 3, 4} (Applying)

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \\ &= \sum_{n=0}^{\infty} x(n) z^{-n} = x(0)z^{-0} + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} \\ &= 1z^{-0} + 2z^{-1} + 3z^{-2} + 4z^{-3} \end{aligned}$$

ROC is entire z plane except z = 0.

66. Determine z transform and roc of the signal {1, 2, 3, 4}(Applying)

$$\begin{aligned} X(z) &= \sum_{n=-\infty}^{\infty} x(n) z^{-n} \\ &= \sum_{n=-3}^{\infty} x(n) z^{-n} = x(-3)z^3 + x(-2)z^2 + x(-1)z^1 + x(0) \\ &= 4 + 3z^1 + 2z^2 + 1z^3 \end{aligned}$$

ROC is entire z plane except z=∞

67. Determine z transform and roc of the signal {1, 2, 3, 4} (Applying)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=-1}^2 x(n) z^{-n} = x(-1) z^1 + x(0) z^0 + x(1) z^{-1} + x(2) z^{-2}$$

$$= 1 z^1 + 2 + 3 z^{-1} + 4 z^{-2}$$

ROC is entire z plane except $Z = \infty, 0$

68. Find the z transform and roc of $a^n u(n)$ (Applying)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} a^n z^{-n} = 1 / (1 - az^{-1}) \text{ roc } |z| > a.$$

69. Find the z transform and roc of $-a^n u(-n-1)$ (Applying)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$X(z) = - \sum_{n=-\infty}^{-1} a^n z^{-n}$$

$$= - \sum_{n=1}^{\infty} (a^{-1}z)^n = 1 / (1 - az^{-1}) \text{ roc } |z| < a.$$

The z-transform of a sequence $x(n)$ is $X(z)$, what is the z transform of $nx(n)$

$$\text{If } z\{x(n)\} = X(z) \text{ then } z\{nx(n)\} = -z d(X(z))/dz$$

70. Find the z-transform of (a) A digital impulse (b) A digital step. (Applying)

(a) Since $x(n)$ is zero except for $n = 0$, where $x(n)$ is 1, we find $x(z) = 1$.

(b) Since $x(n)$ is zero except for $n \geq 0$, where $x(n)$ is 1, we find

$$x(z) = \sum_{n=0}^{\infty} z^{-n} = \frac{1}{1 - z^{-1}}$$

71. What is the relationship between z-transform and DTFT? (Remembering)

The z-transform of $x(n)$ is given by

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} ; \text{ where } z = re^{j\omega} \quad \dots\dots\dots (1)$$

Substituting z in $x(z)$ we get,

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) r^{-n} e^{-j\omega n} \quad \dots\dots\dots (2)$$

The Fourier transform of $x(n)$ is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \quad \dots\dots\dots (3)$$

Equation (2) and (3) are identical, when $r = 1$.

In the z-plane this corresponds to the locus of points on the unit circle $|z| = 1$. Hence $X(e^{j\omega})$ is equal to $H(z)$ evaluated along the unit circle, or $X(e^{j\omega}) = x(z) \Big|_{z=e^{j\omega}}$ For $X(e^{j\omega})$ to exist, the ROC of $x(z)$ must include the unit circle.

UNIT 2

DISCRETE FOURIER TRANSFORM

72. The N point DFT of a sequence x (n) is (Remembering)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i2\pi kn/N} \quad k = 0, 1, 2, \dots, N-1$$

73. The N point IDFT of a sequence X (k) is (Remembering)

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad n = 0, 1, \dots, N-1$$

74. List out the properties of DFT. (Remembering)

i) Periodicity

If X (k) is N point DFT of a finite duration sequence x (n) then

$$x(n + N) = x(n) \text{ for all } n$$

$$X(k + N) = X(k) \text{ for all } k$$

ii) Linearity

If $X_1(k) = DFT[x_1(n)]$ and

$$X_2(k) = DFT[x_2(n)],$$

Then

$$DFT [a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(k) + a_2 X_2(k)$$

iii) Time Reversal of a sequence

$$\text{If } DFT [x(n)] = X(k)$$

$$DFT [x((-n))_N] = DFT[x(N - n)] = X((-k))_N = X(N - k)$$

iv) Circular time shifting of a sequence

$$\text{If } DFT[x(n)] = X(k)$$

Then

$$DFT[x((n - l))_N] = X(k) e^{-j2\pi kl/N}.$$

75. Define DFT. (Remembering)

The DFT is used to convert finite discrete time sequence $x(n)$ to an N – point frequency domain sequence denoted by $X(K)$. The N point DFT of a finite duration sequence $x(n)$ of length (L) . Where $L \leq N$ is defined as,

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-i2\pi kn / N} \quad k = 0, 1, 2, \dots, N-1$$

76. Define IDFT. (Remembering)

The IDFT is used to convert the N – point frequency domain sequence $X(k)$ to an N point time domain sequence. The IDFT of the sequence $X(k)$ of length N – defined as,

$$x(n) = \frac{1}{N} \sum X(k) e^{j2\pi kn / N} \quad n = 0, 1, \dots, N-1$$

77. What is the relation between DTFT and IDFT? (Remembering)

Let $x(n)$ be a discrete time sequence. Now DTFT $[x(n)] = X(\omega)$ and DFT $[x(n)] = X(K)$. The $X(\omega)$ is periodic continuous function of ω and $X(K)$ is periodic sequence. The N point sequence $X(K)$ is actually N samples of $X(\omega)$ which can be obtained by sampling one period $X(\omega)$ at equal intervals.

78. What is the draw back in Fourier transform and how it is overcome? (Remembering)

The draw back in Fourier transform is that it is a continuous function of ω which and so it cannot processed by digital system. This drawback is overcome by using discrete Fourier transform. The DFT converts the continuous function of ω to a discrete function of ω .

79. Give the application of DFT. (Remembering)

- i) The DFT is used for spectral analysis of signals using a digital computer.
- ii) The DFT is used is used to perform filtering operations on signal using digital computer.

80. Distinguish between Fourier series and Fourier transform? (Remembering)

Fourier series expands the signal in terms of sinusoidal orthogonal basis functions. Any periodic signal can be expressed in terms of infinite number of sine and cosine terms. Fourier transform converts the signal from time domain to frequency domain. Fourier transform is mainly used for non periodic signals.

81. What do you understand by periodic convolution? (Remembering)

Let $X_1(n)$ and $X_2(n)$ be two periodic sequences having Fourier co-efficient of $C_1(K)$ and C_2 respectively. Let these co-efficient be multiplied to give $C_3(k)$.

(i.e) $C_3(k) = C_1(k) \cdot C_2(k)$

If $C_3(k)$ are the Fourier Co-efficient of the sequence $X_3(n)$, Then

$$X_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2(n-m)$$

Thus the $X_3(n)$ is the convolution of $X_1(n)$ and $X_2(n)$. In other words, multiplication of the Fourier co-efficient is equivalent to convolution of the corresponding sequences. Since the sequence $X_3(n)$ is also periodic, the convolution is performed only over N samples. Therefore it is called periodic convolution.

82. What is circular convolution? (Remembering)

The convolution of two periodic sequences with periodicity N is called circular convolution. If $X_1(n)$ and $X_2(n)$ are two periodic sequences with N samples in a period, then the circular convolution of $X_1(n)$ and $X_2(n)$ is defined as,

$$X_3(m) = X_1(n) * X_2(n) = \sum_{n=0}^{N-1} X_1(n)X_2(m-n)$$

83. Distinguish between linear and circular convolution? (Remembering)

| S.No. | LINEAR CONVOLUTION | CIRCULAR CONVOLUTION |
|-------|--------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. | If $x(n)$ is a sequence of L number of samples and with M number of samples, after convolution $y(n)$ will contain $N = L+M-1$. | If $x(n)$ is a sequence of L number of samples $h(n)$ with M number of samples, after convolution $y(n)$ will contain $N = \text{Max}(L,M)$ samples. |
| 2. | Linear convolution can be used to find the response of a linear filter | Circular convolution cannot be used to find the response of a linear filter. |
| 3. | Zero padding is not necessary to find the response of a linear filter. | Zero padding is not necessary to find the response. The response of a filter. |

84. What is FFT? (Remembering)

The FFT (Fast Fourier Transform) is a method or algorithms for computing the DFT with reduced number of calculations. The computational efficiency is achieved

by employing divide and conquers approach. This is based on the decomposition of an N point DFT into successively smaller DFTS.

In an N point sequence, if N can be expressed as $N = r^m$, then the sequence can be decimated into r – point sequences. For each r point sequence, r point DFTS are computed. From the results of r point DFTS the r^2 – point DFTS are computed. From the results of r^2 – point DFTS are computed and so on, until we get r^m point DFT. Hence the number of stages of computation is m. The number is called the radix of the FFT algorithm.

85. What is Radix 2 FFT? (Remembering)

The FFT algorithm is most efficient in calculating N point DFT. If number of output points N can be expressed as a power of 2, that is $N = 2^M$, Where M is an integer, then this algorithm is known as radix 2 FFT algorithm.

86. How many multiplications and additions are required to compute N – point DFT using radix 2 FFT? (Remembering)

For performing radix – 2 FFT, the value of N should be such that, $N = 2^M$. The total number of complex addition are $N \log_2 N$ and the total number of multiplications are $(N/2) \log_2 N$.

87. What is the speed improvement factor in calculating 64 point DFT of a sequence using direct Computation and FFT algorithms? (Remembering)

The number of complex multiplications required using direct computation is

$$N^2 = 64^2 = 4096.$$

The number of complex multiplications required using FFT is,

$$N/2 \log_2 64 = 192$$

Speed improvement factor = $4096 / 192$.

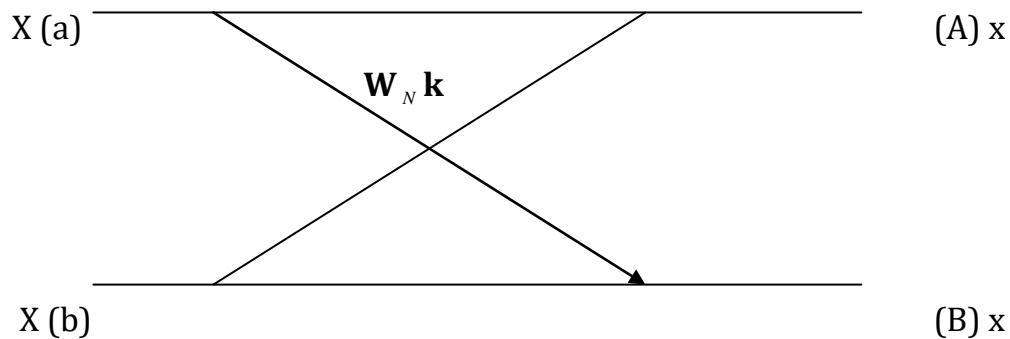
88. What is DIF radix-2 FFT? (Remembering)

The DIT (decimation in frequency) radix -2 FFT is an efficient algorithm for computing DFT. In this algorithm the N – point time domain sequence is converted to two numbers of N/2 point sequences. Then each N/2 point sequence is converted until we get N/2 numbers of 2 point sequences.

No the 2 point DFTS of N/2 numbers of 2 point sequences will give N samples, which is N point DFT of the time domain sequence. Here the equations for forming N/2 point sequence, N/4 point sequences, etc... are obtained by decimation of frequency domain sequences. Hence this method is called DIF.

89. Why FFT is needed? (Remembering)

The FFT is needed to compute DFT with reduced number of calculations. The DFT is required for spectrum analysis and filtering, correlation operations on the signals using digital computers.

90. Draw the butterfly diagram for DIF - FFT algorithm? (Remembering)**91. What are the applications of FFT algorithm? (Remembering)**

The applications of FFT algorithm includes,

- i) Linear Filtering
- ii) Correlation
- iii) Spectrum analysis

92. What is the main advantage of FFT? (Remembering)

FFT reduces the computation time required to compute discrete Fourier Transform.

93. Why linear convolution is important in DSP? (Remembering)

The response or output of LTI discrete time system for any input $x(n)$ is given by linear convolution of the input $x(n)$ and the impulse response $h(n)$ of the system.

94. What is Zero padding? Why it is needed? (Remembering)

Appending zeros to a sequence in order to increase the size or length of the sequence is called zero padding. In circular convolution, when the two input sequence are of different size, then they are converted to equal to size by zero padding.

95. Why circular convolution is important in DSP? (Remembering)

The discrete Fourier Transform (DFT) is used for the analysis and sign of discrete time systems using digital computers. The DFT supports only circular convolution. Hence when DFT techniques are employed, the results of linear convolution are obtained only via circular convolution.

96. What is sectioned convolution? (Remembering)

In linear convolution of two sequences, if one of the sequences is very much larger than the other, then it is very difficult to compute the linear convolution using DFT. In such cases, the longer sequence is sectioned into the size of smaller sequence. Then the linear convolution of each section are combined to get the overall output sequence. This technique of convolution is called sectioned convolution.

97. What are the two methods of sectioned convolution? (Remembering)

The two methods of sectioned convolutions are overlap add method and overlap save method.

98. How will you obtain linear convolution from circular convolution? (Remembering)

Consider two finite duration sequences $x(n)$ and $h(n)$ of duration L samples and M samples respectively. The linear convolution of these two sequences produces an output sequence of duration $L + M - 1$ samples, where as, the circular convolution of $x(n)$ and $h(n)$ give n samples where $N = \text{Max}(L, M)$

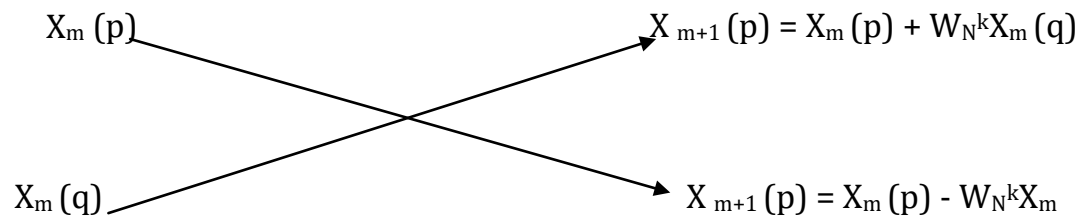
In order to obtain the number of samples in circular convolution equal to $L + M - 1$, both $x(n)$ and $h(n)$ must be appended with appropriate number of zero valued samples. In other words by increasing the length of the sequence $x(n)$ and $h(n)$ to $L + M - 1$ points and then circularly convolving the resulting sequences we obtain the same result as that of linear convolution.

99. What are the differences and similarities between DIF and DIT algorithms? (Remembering)**Difference:**

- ✓ For DIT, the input is bit reversed while the output is in natural order, where as for DIF, the input is in natural order while the bit is reversed.
- ✓ The DIF butterfly is slightly different from the DIT butterfly, the difference being that the complex multiplication takes place after the add - subtract operation.

Similarities:

- ✓ Both algorithms require same number of operations to compute the DFT, both algorithms can be done in place and both need to perform but reversal at some place during the computation.

100. Draw the butterfly diagram for DIT - FFT algorithm? (Remembering)**101. Calculate DFT of the sequence $x(n) = \{1, 1, 2, 2\}$ (Applying)**

$$x(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad K=0,1,2,3,\dots,N-1$$

$$x(k) = \sum_{n=0}^3 x(n) e^{-j2\pi nk/4} \quad K=0,1,2,3$$

$$N=4$$

$$= x(0) + x(1) e^{-jk\pi/2} + x(2) e^{-jk\pi} + x(3) e^{-j3k\pi/2}$$

$$= 1 + e^{-jk\pi/2} - 2e^{-jk\pi} - 2e^{-j3k\pi/2} \quad K=0, 1, 2, 3$$

UNIT 3

DESIGN OF IIR FILTERS

102. What is filter? (Remembering)

Filter is a frequency selective device, which amplifies particular range of frequencies and attenuate particular range of frequencies.

103. What are the types of digital filter according to their impulse response? (Remembering)

IIR (Infinite impulse response) filter

FIR (Finite impulse response) filter

104. Define IIR filter? (Remembering)

The filter designed by considering all the infinite samples of impulse response are called IIR filter.

105. What are the different types of filters based on impulse response? (Remembering)

Based on impulse response the filters are of two types

1. IIR filter
2. FIR filter

The IIR filters are of recursive type, where by the present output sample depends on the present input, past input samples and output samples.

The FIR filters are of non recursive type, where by the present output sample depends on the present input, past input sample and previous input samples.

106. What are the different types of filters based on frequency response? (Remembering)

1. Low pass filter
2. High pass filter
3. Band pass filter
4. Band reject filter

107. State the structure of IIR filter? (Remembering)

IIR filters are of recursive types whereby the present o/p sample depends on present i/p, past i/p samples. The design of IIR filter is realizable and stable. The impulse response $h(n)$ for a realizable filter is. $h(n) = 0$ for $n \leq 0$

108. State the advantage of direct form II structure over direct form I structure. (Remembering)

In direct form II structure, the number of memory locations required is less than that of direct form I structure.

109. How can design digital filters from analog filters? (Remembering)

- Map the desired filter specification into those for an equivalent analog filter.
- Derive the analog transfer function for the analog prototype.
- Transform the transfer of the analog prototype into an equivalent digital filter transfer function.

110. Mention the procedures for digitizing the transfer function of an analog filter. (Remembering)

- Impulse invariance method
- Bilinear transformation method.

111. What do you understand by backward difference? (Remembering)

One of the simplest methods for converting an analog filter into a digital filter is to approximate the differential equation by an equivalent difference equation.

$$\frac{d}{dt}y(t) = y(nT) - y(nT - T)/T$$

The above equation is called backward difference equation.

112. What is meant by impulse invariant method of designing IIR filter? (Remembering)

In this method of digitizing an analog filter, the impulse response of resulting digital filter is a sampled version of the impulse response of the analog filter. The transfer functions of analog filter in partial fraction form.

113. Give the bilinear transform equation between S plane & z plane. (Remembering)

$$S = \frac{2}{T}(1 - Z^{-1} / 1 + Z^{-1})$$

114. What is bilinear transformation? (Remembering)

The bilinear transformation is a mapping that transforms the left half of S plane into the unit circle in the Z plane only once, thus avoiding aliasing of frequency components.

The mapping from the s plane to the Z plane is in bilinear transformation is

$$S = \frac{2}{T} (1 - Z^{-1} / 1 + Z^{-1})$$

115. What are the properties of bilinear transformation? (Remembering)

The mapping for the bilinear transformation is one to one mapping that is for every point Z, there is exactly one corresponding point S, and vice versa.

The $j\Omega$ - axis maps on to the unit circle $|z|=1$, the left half of the s plane maps to the Interior of the unit circle $|z|=1$ and the half of the S plane maps on to the exterior of the unit circle $|z|=1$ and the half of S - plane maps on to the exterior of the unit circle $|z|=1$.

116. Write short notes on pre warping. (Remembering)

The effect of the non linear compression at high frequencies can be compensated. When the desired magnitude response is piece wise constant over frequency, this compression can be compensated by introducing a suitable pre - scaling or pre warping the critical frequencies by using the formula.

117. What are the advantage & disadvantage of bilinear transformation? (Remembering)**Advantage**

- The bilinear transformation provides one to one mapping.
- Stable continuous system can be mapped into realizable, stable digital system.
- There is no aliasing.

Disadvantage:

- The mapping is highly non linear producing frequency, compression at high frequencies.
- Neither the impulse response nor the phase response of the analog filter is preserved in a digit filter obtained by linear transformation.

UNIT 3

DESIGN OF FIR FILTERS

118. What are FIR filter? (Remembering)

The filter designed by selecting finite number of impulse response $h(n)$ obtained from inverse Fourier transform of desired frequency response $H(\omega)$ are called FIR filters.

119. How phase distortion and delay distortion are introduced? (Remembering)

The phase distortion is introduced when the phase characteristics of a filter are nonlinear with in desired frequency band. The delay distortion is introduced when the delay is not constant with in the desired frequency band.

120. Write the steps in FIR filter design. (Remembering)

- ✓ Choose the desired frequency response $H_d(\omega)$
- ✓ Take the inverse Fourier transform and obtain $H_d(n)$
- ✓ Convert the infinite duration sequence $H_d(n)$ to $h(n)$.
- ✓ Take Z Transform of $h(n)$ to get $H(z)$.

121. What are advantages of FIR filter? (Remembering)

- ✓ Linear phase FIR filter can be easily deigned.
- ✓ Efficient realization of FIR filter exists as both recursive and non – recursive structures.
- ✓ FIR filter realize non recursive realization is stable.
- ✓ The round off noise can be made small in non recursive realization of FIR filter.

122. What is the disadvantage of FIR filter? (Remembering)

- ✓ The duration of impulse response should be large to realize sharp cutoff filters.
- ✓ The non integral delay can lead to problems in some signal processing applications.

123. What is the necessary and sufficient condition for the linear phase characteristic of FIR filter? (Remembering)

The phase function should be a linear function of ω , which in trun requires contant group delay and phase delay.

124. List the well known design technique for linear phase characteristic of a FIR filter design? (Remembering)

- ✓ Fourier series method and window method.
- ✓ Frequency sampling method.
- ✓ Optimal filter design method

125. What are the advantages and disadvantage of FIR filters? (Remembering)**Advantage**

1. FIR filters have exact linear phase.
2. FIR filters are always stable.
3. FIR filters can be realized in both recursive and non recursive structure.
4. Filters with any arbitrary magnitude response can be tackled using FIR sequence.

Disadvantage

1. For the same filter specifications the order of FIR filter design can be as high as 5 to 10 times that in an IIR design.
2. Large storage requirement is requirement.
3. Powerful computational facilities required for the implementation.

126. What are the design techniques of designing FIR filters? (Remembering)

There are three well known methods for designing FIR filters with linear phase. They are

1. Window method
2. Frequency sampling method
3. Optimal or minimax design.

127. Distinguish between FIR filter and IIR filters (Remembering)

| S.No | FIR Filter | IIR Filter |
|------|-----------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| 1. | These filters can be easily designed to have perfectly linear phase. | These filters do not have linear phase. |
| 2. | FIR filters can be realized recursively and non recursively. | IIR filters are easily realized recursively. |
| 3. | Greater flexibility to control the shape of their magnitude response. | Less flexibility, usually limited to specific kind of filters. |
| 4. | Errors due to round off noise are less severe in FIR filters, mainly because feed back is not used. | The round off noise in IIR filters is more. |

128. What is Gibb's phenomenon? (Remembering)

One possible way of finding an FIR filter that approximates $H(e^{j\omega})$ would be to truncate the infinite Fourier series at $n = \pm(N-1/2)$. Direct truncation of the series will lead to fixed percentage overshoots and undershoots before and after an approximated discontinuity in the frequency response.

129. List the steps involved in the design of FIR filters using windows. (Remembering)

1. For the desired frequency response $H_d(\omega)$, find the impulse response $h_d(n)$ using equation

$$h_d(n) = \frac{1}{2\pi} \int H_d(\omega) e^{j\omega n} d\omega$$

2. Multiply the infinite impulse response with a chosen window sequence $w(n)$ of length N to obtain filter coefficient $h(n)$.

3. Find the transfer function of the realizable filter

$$H(Z) = Z^{-(N-1)} [h(0) + \sum h(n)(Z^n + Z^{-n})]$$

130. What are the desirable characteristics of the window function? (Remembering)

The desirable characteristics of the window are

1. The central lobe of the frequency response of the window should contain most of the energy and should be narrow.
2. The highest side lobe level of the frequency response should be small.
3. The side lobes of the frequency response should decrease in energy rapidly as ω tends to π .

131. Give the equation specifying the following windows. (Remembering)

- a. Rectangular window
- b. Hamming window
- c. Hanning window
- d. Bartlett window

132. What are the necessary and sufficient conditions for linear phase characteristics in FIR filter? (Remembering)

The necessary and sufficient condition for linear phase characteristics in FIR filter is the impulse response $h(n)$ of the system should have the symmetry property.

$$\alpha = \frac{N-1}{2} \quad \& \quad h(n) = h(N-1-n)$$

Where N is duration of the sequence.

133 Define Phase Delay (Remembering)

It is the amount of time delay each frequency component of the signal suffers in going through the filter.

$$\alpha_p = \frac{-\phi(\omega)}{\omega}$$

133 Define Group Delay (Remembering)

It is the average time delay the composite signal suffers at each frequency

$$\alpha_G = \frac{d\phi(\omega)}{d\omega}$$

134 Brief the steps in FIR Filter Design (Remembering)

Step-1:

Choose the desired frequency response $H_d(\omega)$ of the filter.

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega \left(\frac{N-1}{2}\right)} & ; \quad -\omega_c \leq \omega \leq \omega_c \quad \text{or} \quad |\omega| < |\omega_c| \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Step-2:

Take inverse Fourier transform of $H_d(\omega)$ to get $h_d(n)$.

$$h_d(n) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} H_d(\omega) e^{j\omega n} d\omega$$

Step-3:

Convert the infinite duration sequence $h_d(n)$ to a finite duration sequence $h(n)$ by multiplying with suitable window.

$$h(n) = h_d(n) \times w(n)$$

Step-4:

Take Z transform to $h(n)$ to get system function $H(z)$ of the filter.

Step-5:

Structure realization

135 When an FIR filter is said to be antisymmetric. (Remembering)

Filters having only constant group delay and satisfying the following condition is said to be antisymmetric FIR filters.

$$\alpha = \frac{N-1}{2} \quad \& \quad h(n) = -h(N-1-n)$$

UNIT 5

DSP HARDWARE

136. Mention any four important features of a DSP processor? (Remembering)

- 1) DSP processor should have multiple registers so that data exchange from register is fast.
- 2) DSP operations require multiple operands simultaneously. Hence DSP processor should have multiple operand fetch capacity.
- 3) DSP processor should have circular buffers to support circular shift operations.
- 4) The DSP processor should be able to perform multiply and accumulate operations very fast.

137. Define von - Neumann architecture. (Remembering)

General purpose architecture normally has this type of architecture. The architecture shares some memory for program and data. The processor performs instructions fetch, decode and execute the program operations sequentially. In such architecture, the speed can be increased by pipelining. This type of architecture contains common interval address and data bus, ALU, accumulator, I/O devices and common memory for program and data. This type of architecture is not suitable for DSP processor.

138. Define pipelining. (Remembering)

The process of fetching the next instruction while the current instruction is executing is called as pipelining. The pipelining the through put of the system.

139. What are the advanced addressing modes available in a DSP processor? (Remembering)

There are six new advanced addressing modes which are being used in our programmable DSP processor. They are as follows

- 1) Short immediate addressing mode
- 2) Short direct Addressing mode
- 3) Indirect Addressing mode
- 4) Bit reversed Addressing Mode
- 5) Circular Addressing Mode

140. Explain the dedicated MAC unit. (Remembering)

1. Most of the operations in DSP involve array multiplications. The operations such as convolution, correction require multiply and accumulate operations. In such real time applications, the array multiplication and accumulation must be completed before next sample of input comes. This requires very fast implementation of multiplications and accumulation.
2. The dedicated hardware unit called MAC is used. It is called multiplier accumulator (MAC). It is one of the computational units in processor. The complete MAC operation is executed in one clock cycle.

141. Write a short note on circular addressing modes. (Remembering)

With this mode, the data stored in the memory can be read / written in the circular fashion. This increases the utility of the memory. The memory is organized as a circular buffer. The beginning and address of the circular buffer are continuously monitored. If the address of the memory, then it is set at the beginning address of the memory. This is nothing but circular addressing.

142. Explain about bit reversed addressing mode. (Remembering)

For the computation of FFT, the input data is required in bit reversed format. There increase the utility of the memory is organized as a circular buffer. The beginning and the ending address of circular buffer are continuously monitored. If the address exceeds the ending address of the memory, then it is set at the beginning address of the memory, then it is set at the beginning address of the memory. This is nothing but circular addressing.

143. Mention any four important features of TMS 320 C5X processor. (Remembering)

The family of processor has following features.

1. Powerful 16 bit CPU.
2. 20, 25, 35 and 50 ns single cycle instruction execution time for 5V operation. 25, 40 and 50 ns single cycle instruction execution time for 3V operations.
3. 16 X 16 bit multiply / Add operations can be performed in single cycle.
4. 224 K X 16 bit maximum addressable external memory space. This space is divided into 64 K program data and 64 K I/O and 32 K global memories.

143. Explain scaling shifter in TMS 320C5X processor. (Remembering)

The scaling shifter has a 16 bit input connected to the data bus and 32 bit output connected to the ALU. The scaling shifter produces a left shift of 0 to 16 bits on the input data. The other shifters perform numerical scaling bit extraction, extended precision arithmetic and overflow prevention.

144. Discuss the ARAU in TMS320C5X processor. (Remembering)

There is a register file of eight auxiliary registers. These registers are used for temporary data storage. The auxiliary register file (AR0 – AR 7) is connected to the auxiliary register arithmetic unit (ARAU). The contents of the auxiliary registers can be stored in data memory or used as inputs to central arithmetic logic unit (CALU). The ARAU helps to speed up the operation of CALU.

145. Explain the interrupts in TMS320C5X processor. (Remembering)

The TMS 320C5X has four general purpose interrupts, INT 4 – INT1, one reset and non maskable interrupt. Internal interrupts are generated by serial port (RINT & XINT), by the timer (TINT), through software (TRAP, INTR and NMI), RS low has the highest priority followed by NMI and INT4 has lowest priority.

146. Give the assembly language syntax for TMS320C5X processor. (Remembering)

The source statement can contain following four ordered fields

[label] [:] mnemonic [operand list][; comment]

The source statement follows the guidelines as given below:

- i) All the statement should begin with a label, a blank, an asterisk or a semicolon.
- ii) Labels may be placed before the instruction mnemonic on the same line or on the preceding line in the first column.
- iii) Each field must be separated with blanks.

If comments begin in column 1 it must have semicolon or asterisk at its beginning. In other columns, comments can begin with semicolon.

147. What is the purpose of using NORM instruction? (Remembering)

The purpose of using NORM instruction is used to convert the fixed point number to floating point number.

148. Explain IN & OUT instruction. (Remembering)

IN : Reads 16 bit number from input port and stores in the data memory.

OUT : Reads 16 bit number from data memory and writes into output port.

149. Explain the PUSH & POP instruction in detail. (Remembering)

POP : POPs the top of the stack to ACC.

PUSHD : pushes a data memory location to the top of the stack.

POPD : POPs the top of stack of data memory.

150. What are the classifications of digital signal processors? (Remembering)

The digital signal processors are classified as

- ✓ General purpose digital signal processor
- ✓ Special purpose digital signal processors.

151. Give some examples for fixed point DSPs. (Remembering)

TMS 320C50, TMS320C54, TMS320C55, ADSP – 219x, ADSP- 219xx.

152. What are the factors that influence selection of DSPs? (Remembering)

- ✓ Architectural features
- ✓ Execution speed
- ✓ Type of arithmetic
- ✓ Word length

153. What are the applications of DSPs? (Remembering)

Digital cell phones, automated inspection, voice mail, motor control, video conferencing, Noise cancellation, medical imaging, speech synthesis, satellite communication, etc.

154. What is the pipe line depth? (Remembering)

The number of pipe line stages is referred to as the pipeline depth.

155. What are the different stages in pipe lining? (Remembering)

- ✓ The fetch phase
- ✓ The decode phase
- ✓ Memory read phase
- ✓ The execute phase

156. List the various registers used with ARAU. (Remembering)

Eight auxiliary registers (AR0 – AR7)

Auxiliary register pointer (ARP)

Unsigned 16 bit ALU.

157. What are the different buses of TMS 320C5x and their functions? (Remembering)

The C5x architecture has four buses

- i) Program bus (PB)
- ii) Program Address bus (PAB)
- iii) Data read bus (DB)
- iv) Data read address bus

158. What are the elements that the control processing unit of C5x. (Remembering)

- The central arithmetic logic unit (CALU)
- Parallel logic unit (PLU)
- Auxiliary register arithmetic unit (ARAU)
- Memory mapped registers
- Program controller

159. List the on chip peripherals in C5x. (Remembering)

- i) Clock generator
- ii) Hardware timer
- iii) Software programmable wait state generators
- iv) General purpose I/O pins
- v) Parallel I/O ports.
- vi) Serial port interface
- vii) Buffered Serial port
- Viii) Host port interface

160. What are the general purpose I/O pins? (Remembering)

Branch control input (BIO)
External flag (XF)

161. What are the different stages in pipelining? (Remembering)

- The fetch phase
- The decode phase
- Memory read phase
- The execute phase

IMPORTANT FORMULA**Z - Transform****1. One sided Z - Transform:**

If $\{f(n)\}$ is a sequence defined for $n = 0, 1, 2, 3, \dots$, then

$$Z[f(n)] = Z\{f(n)\} = F(z) = \sum_{n=0}^{\infty} f(n)z^{-n}$$

Where 'z' is an arbitrary complex variable.

2. Note: (Two sided (Or) Bilateral Z - Transform)

If $\{f(n)\}$ is a sequence defined for $n = \pm 0, \pm 1, \pm 2, \pm 3, \dots$, then

$$Z\{f(n)\} = \sum_{n=-\infty}^{\infty} f(n)z^{-n} = F(z).$$

3. Note:

If $f_1(t)$ is a function defined for discrete value of 't' when $t = nT$, $n = 0, 1, 2, \dots$, then

$$Z[f_1(t)] = \sum_{n=0}^{\infty} f_1(t)z^{-n} = \sum_{n=0}^{\infty} f_1(nT)z^{-n} = F(z).$$

4. Inverse Z - Transform:

$$Z[f(n)] = F(z), \text{ then } Z^{-1}[F(z)] = f(n) \quad (\text{Or}) \quad Z^{-1}[F(z)] = f(k).$$

5. Initial Value Theorem (IVT)

If $Z[f(n)] = F(z)$, then $f(0) = \lim_{z \rightarrow \infty} F(z)$ $\left(\because f(0) = \lim_{n \rightarrow 0} f(n) \right)$

6. Final Value Theorem (FVT)

If $Z[f(n)] = F(z)$, then $f(\infty) = \lim_{z \rightarrow 1} (z-1)F(z)$ $\left(\because f(\infty) = \lim_{n \rightarrow \infty} f(n) \right)$

7. Important Formulas:

| S.No: | $f(n)$ (Or) $f(k)$ | $Z[f(n)]$ (Or) $Z[f(k)]$ |
|-------|------------------------------|---------------------------------------------|
| 1. | 1 (Or) 1^n | $\frac{z}{z-1}$ |
| 2. | a , where a is constant | $a \frac{z}{z-1}$ |
| 3. | $(-1)^n$ (Or) $(-1)^k$ | $\frac{z}{z+1}$ |
| 4. | $\left(\frac{1}{a}\right)^n$ | $\frac{z}{z-\frac{1}{a}} = \frac{az}{az-1}$ |
| 5. | a^n | $\frac{z}{z-a}$ |
| 6. | a^{n-1} | $\frac{1}{z-a}$ |
| 8. | n | $\frac{z}{(z-1)^2}$ |
| 9. | n^2 | $\frac{z(z+1)}{(z-1)^3}$ |
| 10. | n^k | $-z \frac{d}{dz} z(n^{k-1})$ |
| 11. | $n(n-1)$ | $\frac{2z}{(z-1)^3}$ |
| 12. | e^{an} | $\frac{z}{z-e^a}$ |

| | | |
|-----|--------------------|---------------------------------------------------------------------|
| 13. | e^{-an} | $\frac{z}{z - e^{-a}}$ |
| 14. | na^n | $\frac{az}{(z - a)^2}$ |
| 15. | na^{n-1} | $\frac{z}{(z - a)^2}$ |
| 16. | e^{-iat} | $\frac{ze^{iat}}{ze^{iat} - 1}$ |
| 17. | $\frac{1}{n}$ | $\log\left(\frac{z}{z-1}\right)$ |
| 18. | $a^n \frac{1}{n}$ | $\log\left(\frac{z}{z-a}\right)$ |
| 19. | $\frac{1}{n+1}$ | $z \log\left(\frac{z}{z-1}\right)$ |
| 20. | $\frac{1}{(n-1)}$ | $\frac{1}{z} \log\left(\frac{z}{z-1}\right)$ |
| 21. | $\frac{1}{n!}$ | $e^{\frac{1}{z}}$ |
| 22. | $\frac{a^n}{n!}$ | $e^{\frac{a}{z}}$ |
| 23. | $\frac{1}{(n+1)!}$ | $ze^{\frac{1}{z}} - z$ |
| 24. | $\sin \theta$ | $\frac{z \sin \theta}{z^2 - 2z \cos \theta + 1}, z > 1$ |
| 25. | $\cos \theta$ | $\frac{z(z - \cos \theta)}{z^2 - 2z \cos \theta + 1}, z > 1$ |
| 26. | $r^n \sin \theta$ | $\frac{zr \sin \theta}{z^2 - 2zr \cos \theta + r^2}, z > 1$ |
| 27. | $r^n \cos \theta$ | $\frac{z(z - r \cos \theta)}{z^2 - 2zr \cos \theta + r^2}, z > 1$ |
| 28. | $\sin at$ | $\frac{z \sin at}{z^2 - 2z \cos at + 1}$ |
| 29. | $\cos at$ | $\frac{z(z - \cos at)}{z^2 - 2z \cos at + 1}$ |

| | | |
|-----|-----------------------|---------------------|
| 30. | $\sin \frac{n\pi}{2}$ | $\frac{z}{z^2+1}$ |
| 31. | $\cos \frac{n\pi}{2}$ | $\frac{z^2}{z^2+1}$ |

8. Important Properties:

| S.No: | Property | $f(n)$ | $Z[f(n)]$ |
|-------|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------|
| 1. | Linearity | $af(n) + bg(n)$ | $aF(z) + bG(z)$ |
| 2. | First Shifting | a) $e^{-at} f(t)$ b) $e^{at} f(t)$ | a) $\{F(z)\}_{z \rightarrow ze^{aT}}$ b) $\{F(z)\}_{z \rightarrow ze^{-aT}}$ |
| 3. | Second Shifting | a) $f(n-k)$ (Or) f_{n-k} b) $f(n+k)$ (Or) f_{n+k} | a) $z^{-k} F(z)$ b) $z^k [F(z) - f_0 - f_1 z^{-1} - f_2 z^{-2} - \dots - f_{k-1} z^{-(k-1)}]$ |
| 4. | Scaling in Z-domain | a) $a^n f(n)$ b) $a^{-n} f(n)$ | a) $F\left(\frac{z}{a}\right)$ (Or) $\{F(z)\}_{z \rightarrow z/a}$ b) $F(az)$ (Or) $\{F(z)\}_{z \rightarrow az}$ |
| 5. | Differentiation in Z-domain | $nf(n)$ | $-z \frac{d}{dz} F(z)$ |
| 6. | Unit Sample Sequence | a) $\delta(n) = \begin{cases} 1, & \text{for } n=0 \\ 0, & \text{for } n \neq 0 \end{cases}$ b) $\delta(n-k)$ c) $\delta(n+k)$ | a) 1, ($\because \delta(0) = 1 \& \delta(n) = 0, n > 0$) b) z^{-k} c) z^k |
| 7. | Unit Step Sequence | a) $u(n) = \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$ b) $a^n u(n)$ | a) $\frac{z}{z-1}$, for $ z > 1$ b) $\frac{z}{z-a}$, for $ z > a$ |

| | | | |
|-----|---------------------|----------------------------------------------|-----------------------------------------|
| 8. | Unit Ramp Sequence | $nu(n)$ | $\frac{z^{-1}}{(1-z^{-1})^2}$ |
| 9. | Convolution Theorem | $\sum_{k=0}^n f(k)g(n-k)$ $= f(n) * g(n)$ | $F(z)G(z)$ |
| 10. | Time Reversal | $f(-n)$ | $F(z^{-1}) = F\left(\frac{1}{z}\right)$ |

ALGEBRA

1. $a^0 = 0,$
2. $a^m \times a^n = a^{m+n}$
3. $\frac{a^m}{a^n} = a^{m-n}$
4. $(a^m)^n = a^{mn}$
5. $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
6. $\sin^2 \theta + \cos^2 \theta = 1$
7. $\sin^2 \theta = 1 - \cos^2 \theta$
8. $\cos^2 \theta = 1 - \sin^2 \theta$
9. $\tan \theta = \frac{\sin \theta}{\cos \theta}$
10. $\cot \theta = \frac{\cos \theta}{\sin \theta}$
11. $1 + \tan^2 \theta = \sec^2 \theta$
12. $\tan^2 \theta = \sec^2 \theta - 1$

$$13. \sec^2 \theta - \tan^2 \theta = 1$$

$$14. 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$15. \cot^2 \theta = \operatorname{cosec}^2 \theta - 1$$

$$16. \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

$$17. \sin 0 = 0, \cos 0 = 1, \tan 0 = 0, \sin 180 = 0, \cos 180 = -1, \tan 180 = 0,$$

$$18. \sin 90 = 1, \cos 90 = 0, \tan 90 = \infty, \sin 270 = -1, \cos 270 = 0, \tan 270 = -\infty$$

$$19. \sin(-\theta) = -\sin \theta; \cos(-\theta) = \cos \theta; \tan(-\theta) = -\tan \theta; \cot(-\theta) = -\cot \theta; \sec(-\theta) = \sec \theta$$

$$20. \sin(90 - \theta) = \cos \theta; \cos(90 - \theta) = \sin \theta; \tan(90 - \theta) = \cot \theta; \cot(90 - \theta) = \tan \theta;$$

$$21. \sec(90 - \theta) = \operatorname{cosec} \theta; \operatorname{cosec}(-\theta) = -\operatorname{cosec} \theta; \sin(90 - \theta) = \cos \theta; \cos(90 + \theta) = -\sin \theta$$

$$22. \operatorname{cosec}(90 + \theta) = \sec \theta; \sin(180 - \theta) = \sin \theta; \cos(180 - \theta) = -\cos \theta;$$

$$23. \tan(180 - \theta) = -\tan \theta; \sin(180 + \theta) = -\sin \theta; \cos(180 + \theta) = -\cos \theta;$$

$$24. \tan(180 + \theta) = \tan \theta$$

$$25. \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$26. \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$27. \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$28. \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$29. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A * \tan B}$$

$$30. \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A * \tan B}$$

$$31. \tan\left(\frac{\pi}{4} - A\right) = \frac{1 - \tan A}{1 + \tan A}$$

$$32. \tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$$

$$33. 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$34. 2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$35. 2 \cos A \cos B = \cos (A+B) + \cos (A-B)$$

$$36. 2 \sin A \sin B = \cos (A+B) - \cos (A-B)$$

$$37. \sin (A+B) \sin (A-B) = \sin^2 A - \sin^2 B$$

$$38. \cos (A+B) \cos (A-B) = \cos^2 A - \sin^2 B$$

$$39. \sin 2A = 2 \sin A \cos A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$40. \tan 2A = \frac{2 \tan A}{1 + \tan^2 A}$$

$$41. \sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$42. \sin C - \sin D = 2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$43. \cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$44. \cos C - \cos D = 2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$45. \cos 2A = \cos^2 A - \sin^2 A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$